

Pulse propagation in interacting one dimensional Bose liquid

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We study wave propagation in interacting Bose liquid, where the short range part of the interaction between atoms is of a hard core type, and its long range part scales with a distance as a power law. The cases of Coulomb, dipole-dipole and Van der Waals interaction are considered. We employ a hydrodynamic approach, based on the exact solution of Lieb-Liniger model, and study the evolution of a density pulse instantly released from potential trap. We analyze semi-classical Euler and continuity equations and construct the corresponding Riemann invariants. We supplement our analysis with numerical calculations and discuss experimental applications for ultracold atom experiments.

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I. INTRODUCTION

Study of interacting one dimensional systems experienced its renaissance in the last decade. A significant progress was made both on the experimental and the theoretical sides. The experimental developments were boosted by new techniques in measurements and fabrication, in the solid state¹ and ultra-cold atoms³⁻⁵ systems. The later employ optical traps, allowing an unprecedented level of control over the system. Tuning the system in and out of Feshbach resonance determines the strength of inter-atomic interaction, while changing optical traps controls the confining potential. This allows to study the systems both in and out of thermal equilibrium, where many interesting physical effects were observed. Quite remarkably, cold atom experiments can be performed both with fermionic or bosonic constituents. This offers an opportunity to further explore the influence of a quantum statistics on correlated one dimensional transport, and in particular on the Bose-Fermi duality. The later was originally established for equilibrium Luttinger liquids (LL), but was considerably extended in the recent years. This generalization emerged as one dimensional quantum liquids were studied beyond the LL paradigm⁶. The key point of the LL theory is Dirac spectrum of single particle fermionic excitations. This amount to linear hydrodynamic description, and in particular dispersionless wave propagation. A generic single-particle spectrum, on other hand, leads to a non-linear quantum hydrodynamic. In other words, finite curvature of fermionic spectrum generates an interaction terms between the bosonic modes. This to be contrasted with original electrostatic interaction that induces the curvature of the in the bosonic branches of excitations. From the bosonic point of view the picture is absolutely dual, as one can (re)-fermionize the problem. In this case the interaction of the bosonic modes induces the curvature of the fermionic excitations, while the curvature of the bosonic spectrum generate electron-electron interaction⁷.

The emergent Fermi-Bose duality allows for a powerful techniques, that enable to study the kinetic of interact-

ing Fermi or Bose system. Moreover, in the presence of a finite short range interaction, the statistics of the elementary constituencies plays only a minor role, and the system is in a universal Fermi-Luttinger liquid regime⁸. A particular illustrative example, is a problem of a pulse evolution, induced by an instant release of a fluid from a confining potential. Within the semiclassical approximation the density pulse splits into left- and right-moving parts, short time after it was created. These parts separate and move in the opposite directions. If the spectrum of single particle excitations were exactly linear, the shape of the left and right moving pulses would remain constant. However, any finite curvature of the single particle spectrum induces a non-linearity, that leads to a deformation of the pulse and unavoidably its "overturn". While this problem was thoroughly studied for fermionics fluids^{7,9-11}, much remains unknown for the bosonic ones. We address this question in the current work. We show that if the density pulse is sufficiently weak, the system is in a universal Fermi-Luttinger regime. Its evolutions is identical to the one studied in^{7,10}, up to a redefinition of the bare parameters. For pulses, that are larger than the energy scale set by a short range interaction, the bosonic character of elementary particles is important and the system is in a new regime. Below we study the evolution of the system in both limits, in the presence of a generic long range interaction.

The structure of this paper is as follows. We begin our discussion by formulating the microscopic model, building on a Lieb-Liniger model. In Sec. II we develop its hydrodynamic description and construct the corresponding Riemann invariantII A. In Sec.III we analyze the behavior of the pulse evolution in the presence of generic long range interaction. We supplement the analytic arguments with numeric calculation. In Section V we summarize our result and discuss further extensions.

II. HYDRODYNAMIC APPROACH TO LIEB-LINIGER MODEL

We start our analysis by introducing a microscopic model, described by the following Hamiltonian

$$\hat{H} = \sum_{i=1}^N \left(\frac{\hat{p}_i^2}{2m} + V_{\text{ext}}(x_i) \right) + \sum_{i<j=1}^N V_{\text{int}}(x_i - x_j). \quad (1)$$

Here V_{ext} is a confining potential and V_{int} is a two-particle interaction, that consist of a short and a long range parts

$$V_{\text{int}}(x) = g'\delta(x) + V_{\text{lr}}(x). \quad (2)$$

Let us first consider the case where only short range interaction is present (we will relax this assumption later on). In the limit the system is described Lieb-Liniger model^{12,13}

$$\hat{H} = \sum_{i=1}^N \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_i^2} + V_{\text{ext}}(x_i) \right) + g' \sum_{i<j=1}^N \delta(x_i - x_j). \quad (3)$$

It is convenient to parametrize the interaction strength in this model by

$$c = \frac{mg'}{\hbar^2}, \quad (4)$$

that has a dimension of an inverse length. Thus, $c = 0$ limit corresponds to free bosons, while $c \rightarrow \infty$ is the hard-core or Tonks-Girardeau limit. In this case the density correlation function of any order coincide with the one of non-interacting fermions. Even for a finite values of interaction strength c the Lieb-Liniger problem is exactly solvable. The problem of pulse evolution for this model was recently solved in Ref.^{14,15}. Moreover, this limit was analyzed in details within the hydrodynamic approach^{16–18}. Since we are interested in the situation where both long and short range part of interaction are present, we consider the known case of Lieb-Liniger model first.

To develop a unified approach it is convenient to pass into the collective degrees of freedom, "bosonizing" the

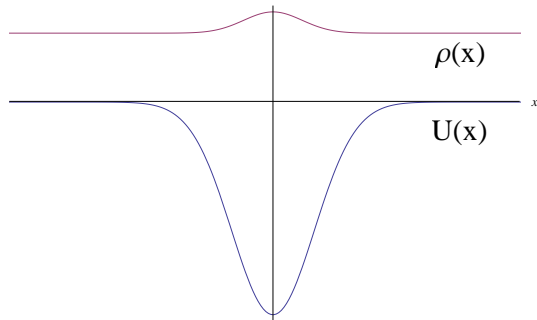


FIG. 1: Setup. Initial density disturbance is created by the application of a potential $U(x)$ which is then switched off at $t = 0$.

bosonic Hamiltonian (1). By doing so, we restrict ourselves to the case of the smooth potential trap. In terms of hydrodynamic variables the Hamiltonian is given by

$$\hat{H} = \int dx \left[\frac{m}{2} \hat{\rho} \hat{v}^2 + E(\hat{\rho}) + \frac{\hat{\rho}_x^2}{8m\hat{\rho}} \right]. \quad (5)$$

Here $\hat{\rho}$ is a density field, \hat{v} is a velocity field and $E(\hat{\rho})$ is an energy per unit volume.

Now on we limit ourselves to quasi-classical accuracy, neglecting the difference between an operator and its expectation value. The equations of motions of Hamiltonian 5 became the continuity and Euler equations for an ideal Bose fluid. The replacement of the true field theory by classical equation of motion, is certainly an approximation. It is justified under a number of conditions: (a) the shape of the pulse is sufficiently smooth; (b) the pulse contains a large number of particles in it; (c) we consider the system at time scales shorter than the inelastic scattering time. (d) the long-range part of the interaction is sufficiently strong. In this work we will focus on the case when all the assumptions above are satisfied.

Assuming that the hydrodynamic fields are slow function of space and time we use an exact solution of Lieb-Liniger model at equilibrium, for a local energy of the fluid per unit volume¹²

$$E(\rho) = \frac{\hbar^2}{2m} \rho^3 e(\gamma). \quad (6)$$

Here

$$\gamma = \frac{c}{\rho} \quad (7)$$

is a dimensionless interaction strength, and the function $e(\gamma)$ is known from the Bethe ansatz solution. It encodes the information about the interaction on a short scale and determines the thermodynamic properties of the liquid. In particular, using 6 one finds the pressure in the fluid

$$P = \frac{\rho^3 e(\gamma)}{m} - \frac{c\rho^2}{2m} \frac{\partial e(\gamma)}{\partial \gamma}. \quad (8)$$

For strong values of interaction strength ($\gamma \rightarrow \infty$), the function $e(\gamma)$ approaches a constant value ($e(\gamma) = \frac{\pi^2}{3}$), reproducing the free fermions limit. For weak values of interaction ($\gamma \leq 1$)

$$e(\gamma) \simeq \gamma \left[1 - \frac{4}{3} \pi \sqrt{\gamma} + \dots \right] \quad (9)$$

in agreement with the Bogoliubov approximation¹³. The Hamiltonian 5 together with Eq.6 lead to the continuity

$$\partial_t \rho + \partial_x(\rho v) = 0 \quad (10)$$

and Euler equation

$$\partial_t v + v \partial_x v + \partial_x w = 0. \quad (11)$$

Here

$$w(\rho) = w_0(\rho_0) - \frac{1}{4}\partial_x^2 \ln \rho - \frac{1}{8}(\partial_x \ln \rho)^2 + \dots \quad (12)$$

is the gradient expansion of enthalpy per unit mass. Its leading part is given by

$$w_0(\rho) = \frac{3\hbar^2}{2m^2}\rho^2 e(\gamma) - \frac{\hbar^2 c \rho}{2m^2} \frac{\partial e(\gamma)}{\partial \gamma}. \quad (13)$$

In the limiting of the strong interaction ($\gamma \gg 1$) one finds

$$w_0 \simeq \frac{\pi^2 \hbar^2 \rho^2}{2m^2} \left(1 - \frac{4}{3\gamma} + \dots\right) \quad (14)$$

In the limit of the weak interaction ($\gamma \ll 1$)

$$w_0 \simeq \frac{\hbar^2 c \rho}{m^2} (1 - \pi\sqrt{\gamma} + \dots) \quad (15)$$

Depending on the average density, the strength of the pulse and interaction the system can be in one of the following regimes. If the amplitude of the pulse is small $(\Delta\rho)^2/2m \ll \rho g' \leq \rho^2/2m$ the system is in the universal Fermi-Luttinger liquid regime. In this case, up to replacement of the parameters (i.e. sound velocity, effective mass and interaction constants) the evolution of the pulse is similar to the one found in a fermionic model¹⁰. If the amplitude of the pulse is big, i.e. case $(\Delta\rho)^2/2m \geq \rho g'$ the system is a qualitatively new regime, where bosonic nature of its constituencies is distinct. In the current work we explore the transition between these two regimes.

We now proceed with the analysis of the pulse evolution, first limiting ourself to the finite range interaction. In this case, as it was shown by Riemann, classical hydrodynamic equation unavoidably leads to the formation of a shock wave. The position of the shock and the behavior of the solution in its vicinity depend on the "regularization". In the absence of long-range interaction the regularization arise due to the terms with high order spatial derivatives in Hamiltonian 5. Away from the shock this derivatives are small, and can be neglected. Close the shock, this term start to play a role, stabilizing the solution, and leading to oscillation in the density and velocity fields. This oscillation takes place of the short scale, indicating that the assumptions of a smooth solution do not hold and hydrodynamic approach is no longer valid. Therefore, it makes it no sense to study the evolution of pulse in the hydrodynamic approach unless long range interaction is present. Before we include this part of the interaction into consideration, let us stay with a Lieb-Liniger model a bit longer and calculate the time when the shock wave is formed and its position. In order to do it, we construct Riemann invariants.

A. Riemann invariants

As it was pointed out by Riemann, a flow of an ideal one dimensional liquid preserves two constants of motion.

The are called Riemann invariants, and are constructed as follows^{19,20}

$$J_{\pm} = v \pm \int \frac{dP}{m\rho s}, \quad (16)$$

where $s = s(\rho)$ is the speed of sound, and the functions P and ρ are related to each other via Eq.(8).

In terms of the Riemann invariants the equation of motion can be written as^{19,20}

$$\left[\frac{\partial}{\partial t} + (v \pm s) \frac{\partial}{\partial x} \right] J_{\pm} = 0. \quad (17)$$

This implies that J_{\pm} are constants along the characteristic curves

$$\frac{dx(t)}{dt} = v(x(t), t) \pm s(x(t), t). \quad (18)$$

Here s is sound velocity, determined by

$$s = \frac{1}{\sqrt{m}} \sqrt{\left(\frac{\partial P}{\partial \rho} \right)_s}, \quad (19)$$

that can be easily evaluated using Eq.(8).

Eq.(18) is highly non-trivial, since it described the motion of the particles with coordinate $x(t)$, moving in an unknown velocity and density fields. However, it greatly simplifies for the problem of a pulse evolution, once its left and right moving are separated in space. As it was rigorously shown by Riemann²⁰, in this case the characteristics are straight lines, see Fig. 2.

Now we construct the Riemann invariant for the Lieb-Liniger model. For the strong interaction limit ($\gamma \gg 1$) the pressure is

$$P(\rho) = \frac{\pi^2 \hbar^2}{3m} \rho^3, \quad (20)$$

that correspond to a gas with a polytropic index $n = 3$. The sound velocity in this limit is

$$s(\rho) = \frac{\pi \hbar}{m} \rho, \quad (21)$$

and the Riemann invariants are

$$J_{\pm} = v \pm \frac{\pi \hbar}{m} (\rho - \rho_0). \quad (22)$$

For the weak interaction the pressure is given by

$$P(\rho) = \frac{c \hbar^2}{2m} \rho^2, \quad (23)$$

corresponding to a gas with the polytropic index $n = 2$. The speed of sound in this limit reads

$$s(\rho) = \sqrt{\frac{c \hbar^2 \rho}{m^2}}, \quad (24)$$

and the corresponding Riemann invariants are

$$J_{\pm} = v \pm \frac{2\hbar\sqrt{c}}{m}(\sqrt{\rho} - \sqrt{\rho_0}). \quad (25)$$

Using this analysis one can easily find the time t_c when the shock wave is formed.

It occurs when two the characteristic curves intersect

$$t_c = \Delta x / \Delta s. \quad (26)$$

Here Δx is the width of the pulse, $\Delta \rho$ is the difference in the density between the pulse and the unperturbed region, Δs is the difference of the sound velocity at the maximum of the pulse and away from it.

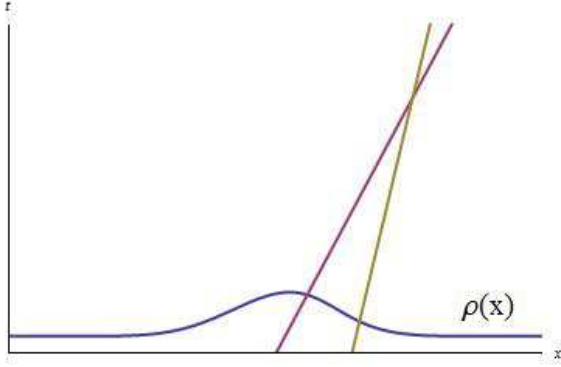


FIG. 2: schematic illustration for the characteristics in a unilaterally propagating pulse

Using Eqs. (21, 24) one finds

$$t_c = \frac{m}{\pi\hbar} \frac{\Delta x}{\pi\Delta\rho}, \quad (27)$$

for $\gamma \gg 1$ and

$$t_c = \frac{\Delta x}{\Delta\rho} \frac{m}{\hbar} \sqrt{\frac{\rho}{c}} \quad (28)$$

for $\gamma \ll 1$. One sees that the time of the shock formation t_c is parametrically longer for weakly interacting Bosons compared with strongly interacting ones.

III. LONG RANGE INTERACTION

We consider three types of long range interaction, that are most relevant for the experiments: Coulomb, dipole and Van der Waals, accounted by

$$H_{\text{int}} = \frac{1}{2} \int dx dx' V(x-x') \rho(x) \rho(x') \quad (29)$$

with a corresponding choice of an interaction potential V . In this case the hydrodynamic equations (10,11) hold, but with an enthalpy term is modified

$$w(x) \rightarrow w(x) + \int dx' V(x-x') \rho(x'). \quad (30)$$

Because all three types of interaction depend on a distance as power, we first discuss a generic power law interaction $V(x) \sim 1/x^\alpha$. A competition between interaction induced dispersion and non-linear terms induces a density modulations, with a characteristic scale Δx . The oscillations are pronounced in the region, where both terms are of the same order

$$v^2 \sim l_0^{\alpha-2} \Delta \rho \Delta x^{1-\alpha}. \quad (31)$$

The relation between the density and the velocity part in the right moving wave can be found from the condition $J_- = 0$, and similarly to the left moving pulse. For the case $\gamma \gg 1$ this estimate yields $v \sim \pi\rho/m$, that leads to the

$$\Delta x \sim \frac{l_0}{(\Delta\rho l_0)^\beta}, \quad \beta = \frac{1}{(\alpha-1)} \quad (32)$$

in agreement with Ref.¹⁰. In the case $\gamma \ll 1$ similar estimate yields $v \sim \Delta\rho\sqrt{c\rho_0}/\rho_0$. The corresponding scale of oscillation is given by

$$\Delta x \sim l_0 \left(\frac{m\rho_0}{c\Delta\rho l_0} \right)^\beta. \quad (33)$$

In a time domain, one expects an appearance of a new pick, that emerges from a bump, approximately each t_c . This estimate holds as long a size of the bump is of the order of the original one. At times when the size and the shape are significantly deformed, this estimate is no longer valid.

We now focus on typical cases of inter-atomic interaction, starting with Coulomb interaction ($\alpha = 1$). We model it by a screened Coulomb potential

$$V(x) = \frac{1}{ml_0} \left(\frac{1}{\sqrt{x^2+a^2}} - \frac{1}{\sqrt{x^2+d^2}} \right). \quad (34)$$

Here a is the transversal size of the trap, and d is the distance to the screening gate. In this case, the long range part of the interaction in the Fourier space is given by

$$V_{\text{lr}}(q) = \frac{2}{ml_0} (K_0(qa) - K_0(qd) - \ln(d/a)). \quad (35)$$

Here K_0 is modified Bessel function, and the effective amplitude of the short range potential is $g = g' + 2/ml_0 \ln d/a$. The renormalization of the bare value of the short range interaction g' occur due to a singularity of Coulomb potential at small distances.

The hydrodynamic equation can be now simulated on the computer, leading to the density evolution shown in the Fig.3. Comparing the evolution of density profile in two limiting cases we note, that the pulse for $\gamma \gg 1$, shown in Fig. 3, has propagated a longer distance than the one with $\gamma \ll 1$. This corresponds to a bigger sound velocity in the former limit as expected. However, the difference in the distance, that the pulse has propagated,

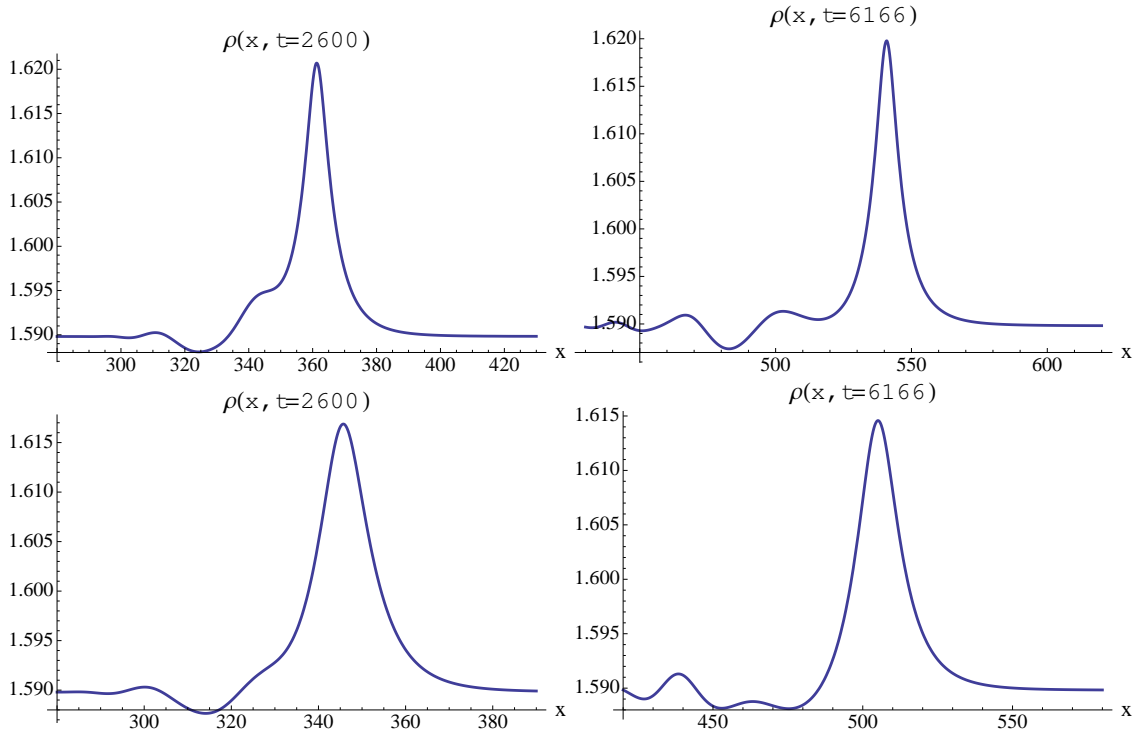


FIG. 3: Density profile after the shock for the case of Coulomb interaction, $\gamma \gg 1$ (top) and $\gamma \ll 1$ (bottom).

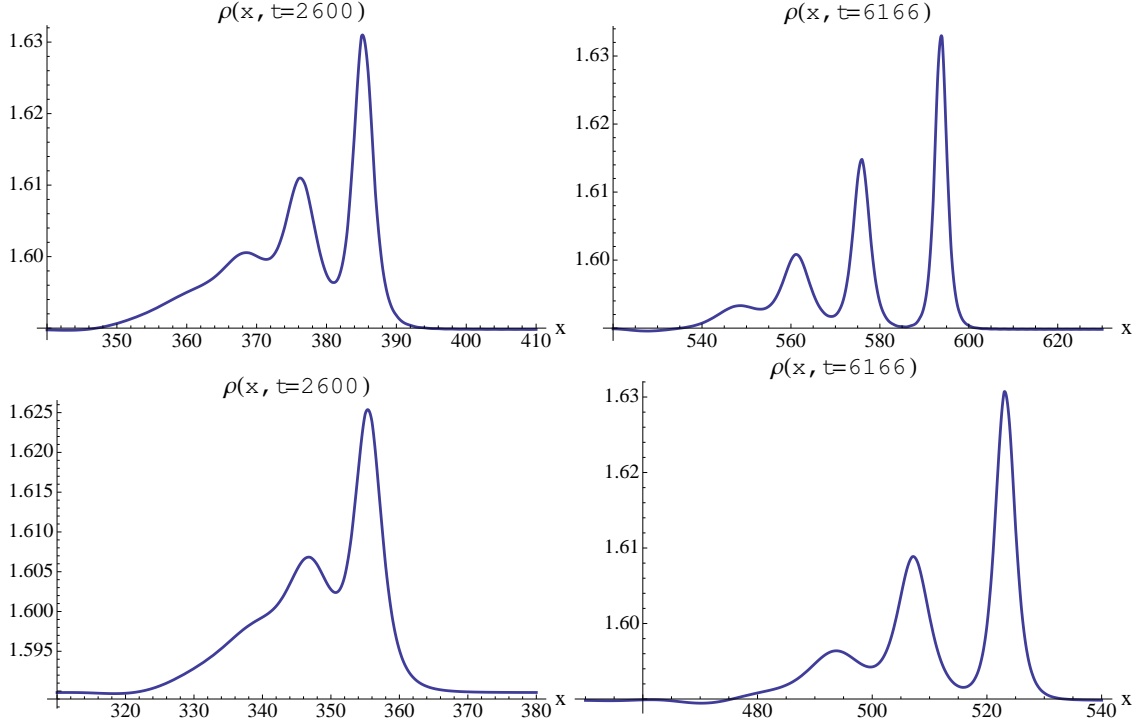


FIG. 4: Density profile after the shock for the case of dipole interaction, $\gamma \gg 1$ (top) and $\gamma \ll 1$ (bottom)

and its shape is rather mild. This has to do with a strong renormalization of the bare value of a short range interaction (g'), by a Coulomb potential. The difference be-

tween the renormalized values of short range interaction g is not big. As a result, for Coulomb interaction the system remains effectively in the Fermi-Luttinger limit, even

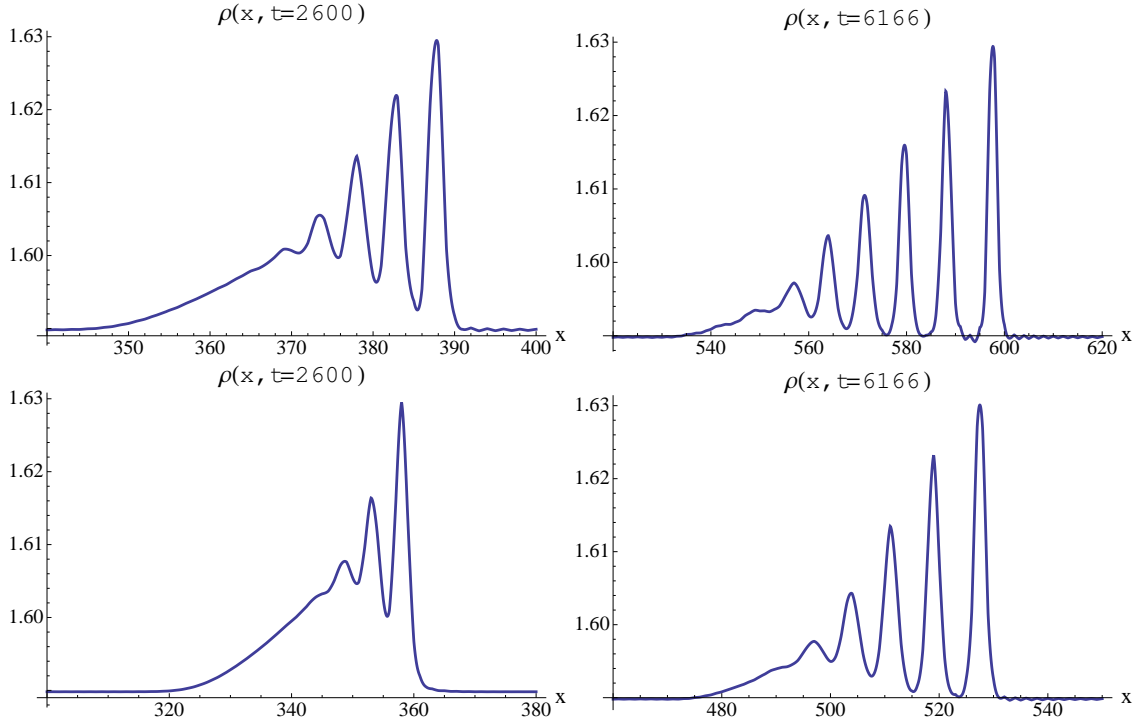


FIG. 5: Density profile after the shock for the case of Van der Waals interaction, $\gamma \gg 1$ (top) and $\gamma \ll 1$ (bottom)

though it was not a case for original Lieb-Liniger model. The main pick in Fig.3 is followed by an oscillatory tail.

If atoms are neutral, the direct Coulomb interaction we have just discussed is absent. In this case, the leading part of a long range interaction is determined by dipole-dipole interaction ($\alpha = 3$). Known examples are ultra-cold chromium^{21,22} and dysprosium²³ atoms. Tuned to a state with a non-zero dipole moment these atoms interact via

$$V(x) = \frac{C_3}{(|x| + a)^3}. \quad (36)$$

In a Fourier space it corresponds to

$$V_{\text{lr}}(q) = C_3 q^2 \ln(q a e^{\gamma_E}), \quad (37)$$

and $g = g' + \frac{C_3}{a^2}$ for the renormalized short range interaction strength; $\gamma_E \simeq 0,5772$ is Euler constant. The evolution of the pulse in this case is shown in Fig. 4 for the values of weak and strong values of a bare short range interaction. One notice a substantial difference between the evolution of the liquid with small and large values of bare interaction. In the later case, the pulse moves with a higher speed and the number of oscillations in it is larger.

Finally, if atoms do posses a dipole moment, but its direction is not polarized the long range part of interaction is of Van der Waals type ($\alpha = 6$). This is a situation where majority of contemporary cold gases belong. Because this interaction is rather weak, it hardly influences a scattering length l_0 . For this reason this interaction is

usually neglected. However, it play a major role for a dispersive regularization of the shock waves, as we show next. In the real space the of Van der Waals interaction decays as

$$V(x) = -C_6/(x^2 + a^2)^3. \quad (38)$$

Passing to q space one finds

$$V_{\text{lr}}(q) = +\frac{\pi C_6 q^2}{8a^3} \quad (39)$$

and $g = g' - \frac{3\pi C_6}{8a^3}$. The results of pulse evolution in this case are depicted in Fig.5. The difference between $\gamma \ll 1$ and $\gamma \gg 1$ in this case is stronger pronounced here than for a dipole interaction, we discussed earlier. The trend however remains the same. The number of picks in the oscillatory tail increases with increasing γ , while the amplitude of the leading pick remains practically unchanged. One may note, that Van der Waals interaction induces a regularization that has the same form as one induced by a quantum pressure. However, its coefficient is controlled by the interaction strength and does not vanish in large density limit. As a result, as long as a condition $\Delta\rho \ll \rho$ is satisfied Van der Waals interaction regularizes shocks more efficiently than the quantum pressure. Interaction decays with distance slower than $1/x^6$ induces a regularization with a smaller number of spatial derivatives. In this case it dominates the quantum pressure term even stronger.

In addition to density evolution, hydrodynamic equations contain the information about the velocity of

quantum fluid in space and time. It is quite reasonable to discuss it as well. This is what we do in the next section, in the context of interferometry measurements in Bose liquid.

IV. INTERFERENCE MEASUREMENTS

The interference measurements probe phase correlation of the Bose-Einstein condensate^{24–29}. The major steps of such experiment are: (1) a split of a condensate into two parts, accomplished by changing of a confining potential or by controlling an internal degree of freedom; (2) an independent evolution of two parts for a period of time; (3) an overlap of two parts. The latter creates a matter-wave interference pattern. This pattern is usually understood in terms of the condensate wave function

$$\Psi(x, t) = \sqrt{\rho(x, t)} e^{i\theta(x, t)}. \quad (40)$$

The interaction plays a multiple-role in an interferometry. It influences the dynamics of a condensate, as we have already seen in Sections II and III. In addition, interaction also leads to an inelastic scattering that destroys the quantum coherence. Note, that the coherence can be also destroyed by another mechanisms, for example on the stage of when the condensate is split^{30,31}, three-body recombinations³², or by an external noise. In our analysis we assume that these sources of dephasing are weak, and take into account only the processes that are induced by the density pulse.

If short range interaction is strong ($\gamma \gg 1$), i.e. in Tonk-Girardeau limit, the bosonic problem can be mapped onto the fermionic one. In this case the zero temperature inelastic rate is given by^{6,8}

$$\frac{1}{\tau_p} \sim [V_0(V_0 - V_{k-k_F})]^2 \frac{(k - k_F)^4}{m^3 v_F^6}, \quad (41)$$

where V_q is the Fourier transform of the interaction between fermionic particles $V(r)$. Estimating the typical momentum scale k as $k - k_F \sim \Delta\rho$, for interaction that scales like $1/r^\alpha$. Here, $1 \leq \alpha < 3$, and the length l_0 parameterizing the strength of the interaction is the Bohr radius for the potential $V(r)$. The inelastic decay rate is thus given by

$$\frac{1}{\tau_p} \sim \frac{1}{m^7 v_F^6 l_0^8} (l_0 \rho_\infty)^{2\alpha-2} (l_0 \Delta\rho)^{2+2\alpha}. \quad (42)$$

On the other hand, the characteristic time scale for the density ripples is given by Eq. 27.

Assuming moderate interaction strength $l_0 \sim 1/\rho_\infty$ one finds¹⁰

$$\frac{t_c}{\tau_p} \sim N \left(\frac{\Delta\rho}{\rho_\infty} \right)^{2\alpha}, \quad (43)$$

where N is a number of particles contained in the pulse. We see that in the limit of small $\Delta\rho/\rho_\infty \ll 1$ the characteristic time τ_p of inelastic decay is much larger than the shock time t_c .

In the limit of weakly interacting Bose gas ($\gamma \ll 1$) the life time of bosonic excitations was calculated in Ref.³³. At zero temperature, the life time of a long-wave boson is given by

$$\frac{1}{\tau_q} \sim A^2 \frac{\hbar^2 \rho_0^2}{m} \left(\frac{q}{q_0} \right)^7, \quad \text{for } q \ll q_0. \quad (44)$$

Here $q_0 = \sqrt{8mg\rho}$ and A is a three-body collision amplitude. It is equal to zero in our model, but is present in a generic case, for example due to excitations of a high energy transversal modes³⁴. For large pulse, with $\Delta\rho \sim q \gg q_0$ the inelastic length was calculated in³⁵, who found

$$\frac{1}{\tau} \sim A^2 \frac{\rho^2}{m}. \quad (45)$$

Since in all limits the shock formation time t_c remain finite within integrable Lieb-Liniger model and does not depend on A these two time scales are independent. Assuming that the integrability is violated only weakly we will neglect the inelastic processes altogether.

We now analyze the emerging interference pattern of the problem of pulse evolution for the limits considered above. To cast in a conventional form we use a "condensate" wave function, Eq.(40). Note, that in one dimension, due to pronounced quantum fluctuation effect the true long range order does not exist. Nevertheless, one still can formally bosonize the bosonic theory^{12,36}, that results in the practically identical model. The bosonic field θ is related to a velocity field in the usual way

$$v(x, t) = \frac{1}{m} \partial_x \theta(x, t). \quad (46)$$

After this transformation both Euler and continuity equation are combined into a single non-local Gross-Pitaevskii equation

$$i\partial_t \Psi(x, t) = \left(-\frac{1}{2m} \partial_x^2 - \mu + V_{\text{ext}}(x, t) \right) \Psi(x, t) + m w_0 (|\Psi(x)|^2 \Psi(x) + \int dx' |\Psi(x', t)|^2 U(x - x') \Psi(x, t)). \quad (47)$$

Here we define

$$U(x) = \int \frac{dq}{2\pi} (V(q) - V(0)) e^{iqx}. \quad (48)$$

Thus the solution of non-local Gross-Pitaevskii equation (47) is fully equivalent to the non-local hydrodynamic equations we studied earlier. The profile of the velocity field are shown in Fig.6 for the case of a dipole interaction. As we see, the profiles of the velocity field is quite similar to the density. This is what we have expected, in our previous estimate. Indeed, it follows from the Riemann invariants for the right (left) moving wave, the functions ρ and v are dependent. Our numerical simulations show that on the whole, this picture holds in

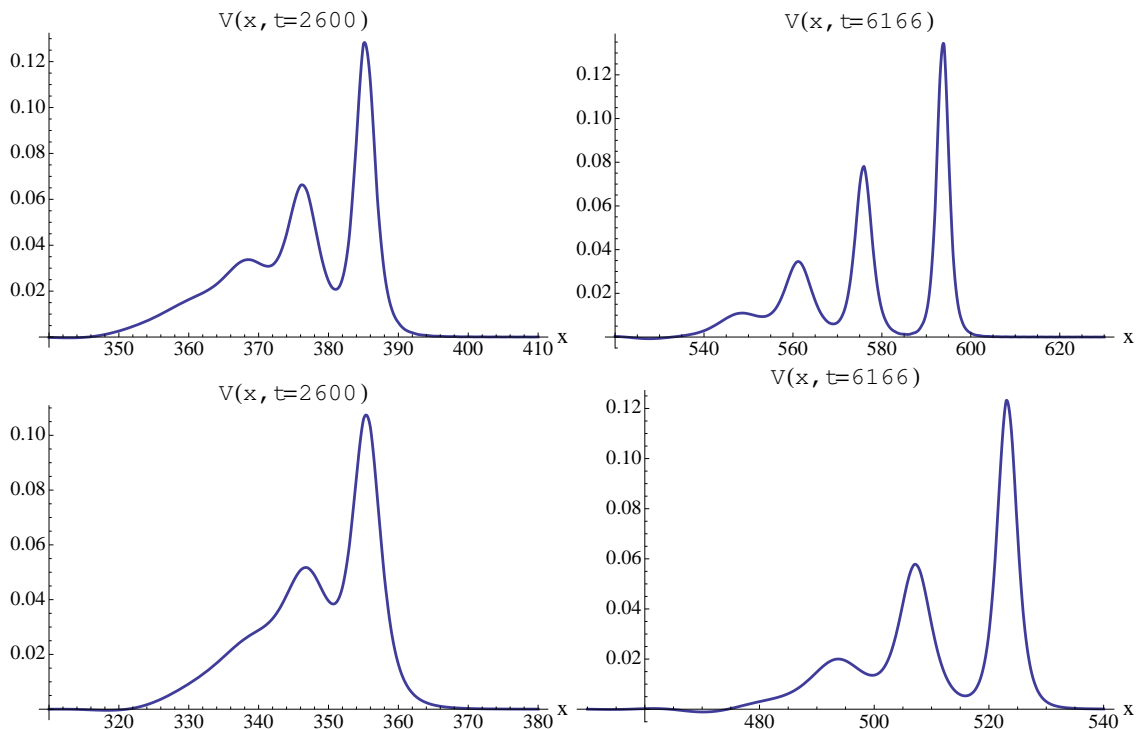


FIG. 6: Profile of the velocity field for dipole interaction $\gamma \gg 1$ (top) and $\gamma \ll 1$ (bottom)

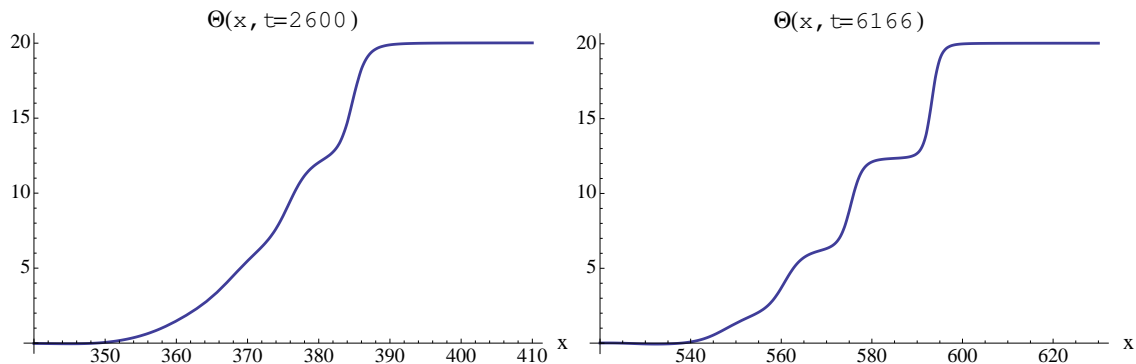


FIG. 7: Profile of the θ field for dipole interaction $\gamma \gg 1$

the presence of a generic long range interaction. As for the velocity field, the oscillations for large values of γ are more pronounced than for small γ . The velocity field v enables us to calculate the field θ , see Fig.7. As we expected, picks of the velocity field correspond to steps in the θ -field. The later lead to oscillation of the "condensate" wave function

$$\text{Re} \left(\Psi(x, t) / \sqrt{\rho(x, t)} \right) = \cos(\theta(x, t)), \quad (49)$$

and the emergence of the interference pattern, shown in Fig. 8. Note, that the phase difference in our case is driven by the non-equilibrium pulse and arises due to a different length particles on crests and troughs of the wave have passed. By controlling the strength of the initial pulse one is able to change the pattern, similarly to

changing a magnetic field in the conventional Aharonov-Bohm interferometers.

V. SUMMARY AND OUTLOOK

In this paper we studied the evolution of a smooth density pulse in a one dimensional Bose liquid, in the presence of short and long range interaction. At the start of the evolution the pulse splits into a left and a right components, that quickly separate and propagate independently. Prior to the shock formation at t_c , the long range part of interaction plays a minor role, while short range part of the interaction determines the enthalpy of the fluid in accordance with the exact solution of Lieb-

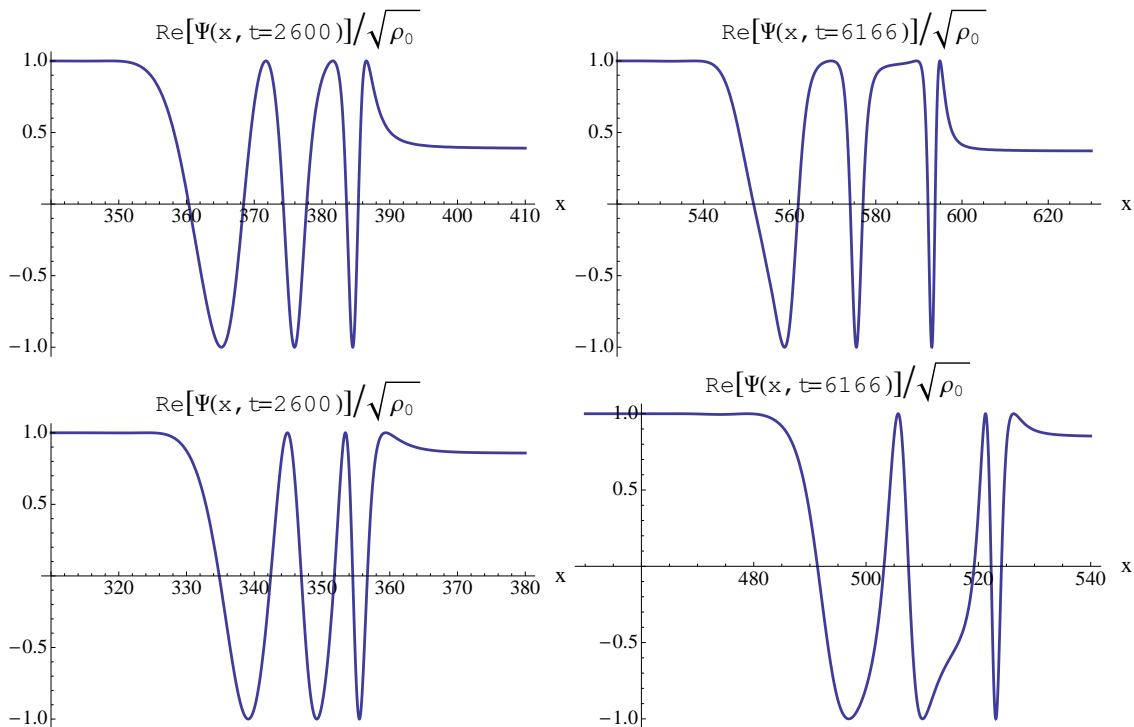


FIG. 8: Interference pattern for dipole interaction $\gamma \gg 1$ (top) and $\gamma \ll 1$ (bottom)

Liniger model. By varying the strength of a short range interaction γ we were able to explore a variety of regimes, from Tonk-Girardeau gas (for $\gamma \gg 1$) to weakly interacting Bose gas (for $\gamma \ll 1$).

With a formation of a shock wave, the long range interaction starts to compete with the non-linear terms, stabilizing the solution. The tug-of-war between the nonlinearity and a long range interaction results in oscillations of density and velocity fields. The scale of these oscillations depends on the type of long range interaction and the strength of the short range potential (γ).

We have computed the evolution of the density pulse in this regime numerically, for the cases of Coulomb, dipole-dipole and Van der Waals interaction. We constructed the Riemann invariants for the problem and compared our numerical results with analytic estimates. We found that the shock formation time (t_c), the period of oscillations and the magnitudes of the velocity and density fields are in agreement with our estimates. We found that in all the cases we considered, the interaction induces a regularization that dominates over the quantum pressure term, in the limit of sufficiently smooth density

pulse. It implies that the form of the pulse in the long time limit is controlled by the long range part of the interaction, even in the limit of Van der Waals interaction.

We have also studied an interference pattern of the matter field, induced by a non-equilibrium pulse. After casting the hydrodynamic equations in terms of non-local Gross-Pitaevskii equation, we computed the phase of the condensate wave function. The oscillations of the velocity field lead to the kinks in this phase. The later manifest itself through interference pattern in time domain. Our predictions can be tested experimentally and we hope that our work will stimulate such experiments.

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